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A METHOD FOR COMPUTING SERIES SYSTEM RELIABILITY WITH UNEQUAL COMPONENT SAMPLE SIZES

by

J. R. Borsting
W. M. Woods

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Monterey, California

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Superintendent

Dr. R. F. Rinehart,
Academic Dean

ABSTRACT:

A method is presented for constructing system reliability using component failure data when the sample sizes for testing on the component parts differ greatly. The procedure can be applied to weapons systems as easily as subsystems. No assumptions about failure distributions are made. The accuracy of the procedure was examined by computer simulations and in this manner the procedure has demonstrated high accuracy for cases of practical interest.

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Prepared by: J. R. Borsting
W. M. Woods

Approved by:

J. R. Borsting
Chairman, Department of
Operations Analysis

Released by:

C. E. Menneken
Dean of
Research Administration

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A METHOD FOR COMPUTING SERIES SYSTEM RELIABILITY WITH UNEQUAL COMPONENT SAMPLE SIZES

1. Summary

A procedure has been constructed for obtaining a lower confidence interval on system reliability using component failure data when component sample sizes differ greatly. Although the authors feel the accuracy of the procedure can be improved, the procedure is surprisingly accurate for cases that are of practical importance to the Navy today, in particular to the Polaris Missile subsystem and the Fleet Ballistic Missile Weapon System.

The procedure appears to have greater accuracy when the number of components is large. The presence of large differences in the sample sizes for the components has little if any affect on the accuracy, and accuracy appears to be very good when sample sizes are realistically moderate and the component reliabilities are near those of interest. The procedure can be as easily applied to an entire weapon system as it can to a subsystem, and has versatility in that several different types of test data may be used. In particular, it can be used in the development phase with the use of K-factors (degrading factors that account for differences in test and flight environments) to obtain early estimates of system reliability. Later in the program qualification test data can also be used either separately or in conjunction with data obtained earlier.

If this procedure were supplemented with Navy OD 28584, the resulting document would be one that could be used to obtain system or subsystem reliability for any weapon system without making any assumption as to the failure distribution of any type of component or subsystem.

2. Description of Procedure

For a device (system, subsystem, major component, etc.) that has k components connected in logical series, its reliability R_s, using the product rule, can be defined by

$$R_s = \prod_{i=1}^{k} p_i \tag{2.1}$$

where p_i is the reliability of component i. It will be understood throughout this report that when we use the term reliability we have in mind a specific mission under some fixed set of environmental conditions. In this sense, we use the term reliability in a generic manner. When applying the procedure, it makes no difference whether the components are continuously operating-type items or cycle-type items. Formula (2.1) is valid in either case. This is also true if the device, hereafter called a system, has some components of each type.

The problem of interest is to obtain a lower 100 (1- [illegible] con-fidence bound for the system reliability R_s using estima[illegible] of the

component reliabilities. In order to do this, we shall need the following notation:

(2.2)

n_i: The number of items of component i that have been put on test and given the opportunity to perform their mission.

f_i: The number of failures for the i^{th} component.

q_i: $1 - p_i$

$\hat{q}_i$: $\frac{f_i}{n_i}$

The procedure for estimating the lower confidence bound is as follows:

(2.3) $$\hat{T}_i = a_i \hat{q}_i + \frac{b_i}{2} \hat{q}_i^2$$

where

(2.4) $$a_i = \frac{2 n_i - 3}{2 (n_i - 1)}, \quad b_i = \frac{n_i}{n_i - 1}$$

Let

(2.5) $$\hat{S} = \sum_1^k \hat{T}_i$$

and

(2.6) $$\hat{r} = \left(\sum_1^k \hat{T}_i\right)^2 \Big/ \left(\sum_1^k \frac{\hat{T}_i}{n_i}\right)$$

Let $\chi^2_{\alpha, n}$ be defined by

(2.7) $$P\left[\chi_n^2 > \chi^2_{\alpha, n}\right] = \alpha .$$

Then the lower 100 $(1-\alpha)$% confidence bound $\hat{R}_s(\alpha)$ for R_s using this procedure is given by

(2.8) $$\hat{R}_s(\alpha) = \exp\left\{ - \frac{\hat{S} [2 \hat{r}]}{\chi^2_{1-\alpha, [2 \hat{r}]}} \right\}$$

where $[2\hat{r}]$ denotes the smallest integer greater than or equal to 2 r. That is, we are asserting that

$$P[R_s \geq \hat{R}_s(\alpha)] = 1 - \alpha \tag{2.9}$$

One way to check the accuracy of this procedure is to assign values to the parameters k, n_i, q_i, find the α-th percentile point in the distribution of $\hat{R}_s(\alpha)$, and compare this point with R_s. If the procedure is exact they should be the same. This was done and the results appear in Tables 1A, 1B, and 1C.

When the sample sizes are all the same, i.e., $n_i = n$, $i = 1, 2, \cdots, k$, the q_i are small and k is not large, a well-known procedure can be used to compute system reliability. This procedure is usually called the Poisson approximation method, and we have evaluated the accuracy of this procedure in the same way as described in the preceding paragraph. In Tables 1A, 1B, and 1C, Y_α is the α-th percentile point of the Poisson approximation method, and $R_{s,\alpha}$ is the α-th percentile of the new procedure suggested here. The confidence intervals obtained by either of these procedures would be exact if the respective Y_α or $R_{s,\alpha}$ equal the corresponding value of R_s in the tables. Thus $R_s - R_{s,\alpha}$ measures the accuracy of the new confidence interval procedure and $R_s - Y_\alpha$ measures the accuracy of the Poisson approximation procedure.

3. Analysis

The analysis needed to support Section 2 of this report will be supplied in the next report for this project. Some additional work needs to be done on this study to

a) supply continuity correction factors to improve the accuracy of the procedure,

b) construct bounds on possible errors to determine bounds on accuracy,

and

c) establish criteria for using this procedure.

TABLE 1A

Simulation Results: $R_{s,\alpha}$ and Y_α for Given k, n_i, p_i ; All n_i Equal

No. of Components in the System	n_i	p_i	R_s	$R_{s,.05}$	$R_{s,.10}$	$R_{s,.20}$	$Y_{.05}$	$Y_{.10}$	$Y_{.20}$
5	10, i=1, . . . , 5	.85, i=1, . . . , 5	.44	.42	.45	.48	.08	.20	[illegible]
5	50, i=1, . . . , 5	.99, i=1, . . . , 5	.951	*1.00	.895	.914	.94	.92	[illegible]
5	100, i=1, . . . , 5	.95, i=1, . . . , 5	.77	.77	.77	.77	(Not available)		
15	20, i=1, . . . , 15	.99, i=1, . . . , 15	.86	.68	.70	.79	.76	.80	[illegible]
15	50, i=1, . . . , 15	.99, i=1, . . . , 15	.86	.80	.83	.86	.85	.84	[illegible]
30	20, i=1, . . . , 30	.99, i=1, . . . , 30	.74	*.60	.66	.71	.68	.67	[illegible]
5	20, i=1, . . . , 5	.85, i=1, . . . , 5	.44	.44	.45	.45	.21	.23	[illegible]
15	100, i=1, . . . , 15	.95, i=1, . . . , 14							
		.85, i=15	.79	.78	.79	.79	.77	.78	[illegible]
15	50, i=1, . . . , 15	.95, i=1, . . . , 14							
		.85, i=15	.79	.77	.78	.79	.76	.76	[illegible]
15	20, i=1, . . . , 15	.95, i=1, . . . , 14							
		.85, i=15	.79	.69	.71	.77	.76	.73	.72

* This error can be corrected somewhat with continuity correction factors.

TABLE 1B

Simulation Results for a System with 15 components

Component Number	n_i	p_i	R_s	$R_{s,.05}$	$R_{s,.10}$	$R_{s,.20}$
1	250	.99				
2	40	.99				
3	120	.99				
4	15	.99				
5	130	.99				
6	65	.99				
7	70	.99				
8	130	.99	.86	.87	.87	.87
9	30	.99				
10	20	.99				
11	75	.99				
12	90	.99				
13	100	.99				
14	60	.99				
15	60	.99				

[illegible]

TABLE 1C

Simulation Results for A System with 13 Components

Component Number	n_i	p_i	R_s	$R_{s,.05}$	$R_{s,.10}$	$R_{s,.20}$
1	150	.995				
2	90	.985				
3	75	.979				
4	100	.988				
5	125	.982				
6	18	.980	.72	.71	.69	.71
7	28	.967				
8	19	.900				
9	5	.980				
10	125	.995				
11	63	.970				
12	125	.995				
13	59	.968				

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